MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The number of rational terms in the binomial expansion of $\left(5^{1/2}+7^{1/8}\right)^{1016}$ is

(1) 129 (2) 128

(3) 127 (4) 130

Answer (2)

Sol. $T_{r+1} = 1016_{C_r} \cdot \left(7^{\frac{1}{8}}\right)^4 \cdot 5^{\frac{1}{2}(1016-r)}$

$\Rightarrow \frac{r}{8}$ and $\frac{1016-r}{2} \in \text{Integer}$

$\Rightarrow 8/r \Rightarrow r = 0, 8, 16, ...1016$

$\Rightarrow r = 0 \times 8, 1 \times 8, ... 127 \times 8$

$\Rightarrow$ Total 128 r such that T_{r+1} is rational

2. $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+..... = \frac{\pi^4}{90}$,

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+ .... = \alpha$

$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+.... = \beta$

Then find $\frac{\alpha}{\beta}$.

(1) 15 (2) 14

(3) 23 (4) 18

Answer (1)

Sol. $\frac{1}{1^4}+\frac{1}{2^4}+\frac{3}{3^4}+\frac{1}{4^4}+....\infty = \frac{\pi^4}{90}$

$$\left(\frac{1}{1^4}+\frac{3}{3^4}+\frac{1}{5^4}+ ....\right)+\left(\frac{1}{2^4}+\frac{1}{4^4}+....\infty\right) = \frac{\pi^4}{90}$$

$$\propto + \frac{1}{2^4}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+....\infty\right) = \frac{\pi^4}{90}$$

$$\propto + \frac{1}{2^4}\cdot\frac{\pi^4}{90} = \frac{\pi^4}{90}$$

$$\propto = \frac{\pi^4}{90}\cdot\frac{15}{16}$$

$$\because \quad \alpha+\beta = \frac{\pi^4}{90}$$

$$\therefore \quad \beta = \frac{\pi^4}{90} - \frac{15\pi^4}{90\times16} = \frac{\pi^4}{90\times16}$$

$$\therefore \quad \frac{\alpha}{\beta} = \frac{\frac{\pi^4}{90}\times\frac{15}{16}}{\frac{\pi^4}{90}\times\frac{1}{16}}$$

$$\therefore \quad \frac{\alpha}{\beta} = 15$$

3. There are 12 points in a plane in which 5 are collinear such that no three of them are in straight line, then the number of triangles that can be formed from any 3 vertices from 12 points.

(1) 220

(2) 210

(3) 230

(4) 240

Answer (2)

Sol.

Number of triangles = ${}^5C_1 \times {}^7C_2 + {}^7C_3 + {}^5C_2 \times {}^7C_1$

= 210

4. Probability of event A is 0.7 and event B is 0.4, $P(A \cap B^C) = 0.5$, then the value of $P(B | A \cup B^C)$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

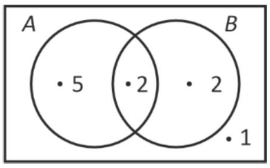
(3) $\frac{1}{4}$ (4) $\frac{3}{4}$

Answer (3)

Sol. $P\left(A \cap B^C\right) = \frac{1}{2}$

$$P\left(\frac{B}{A \cup B^C}\right) = \frac{P\left(B \cap \left(A \cup B^C\right)\right)}{P\left(A \cup B^C\right)}$$

$$= \frac{P\left(\left(A \cap B\right) \cup B \cap B^C\right)}{P\left(A \cup B^C\right)}$$



$$= \frac{P(A \cap B)}{P\left(A \cup B^C\right)} = \frac{0.2}{0.5 + 0.2 + 0.1}$$

$$\frac{0.2}{0.8} = \frac{1}{4}$$

5. $\int_{-1}^{3/2} |\pi^2 x \sin(\pi x)| \, dx.$

(1) $4\pi + 1$ (2) $3\pi + 1$

(3) $5\pi + 1$ (4) $6\pi + 1$

Answer (2)

Sol. $I = \int_{-1}^{3/2} |\pi^2 x \sin(\pi x)| \, dx$

$$= \int_{-1}^{1} |\pi^2 x \sin(\pi x)| \, dx + \int_{1}^{3/2} |\pi^2 x \sin(\pi x)| \, dx$$

$$= 2\int_{0}^{1} |\pi^2 x \sin(\pi x)| \, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)| \, dx$$

$$= 2\pi^2 \int_{0}^{1} |x \sin(\pi x)| \, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)| \, dx$$

$$\because \int x \sin(\pi x) dx = x\left(\frac{-\cos \pi x}{\pi}\right) - \int \frac{-\cos \pi x}{\pi} dx$$

$$= -\frac{x}{\pi} \cos \pi x + \frac{1}{\pi^2} \sin \pi x + C$$

$$\therefore \; I = 2\pi^2\left(\frac{1}{\pi}\right) - \pi^2\left(-\frac{1}{\pi^2} - \frac{1}{\pi}\right)$$

$= 2\pi + 1 + \pi$

$= 3\pi + 1$

6. The product of last 2 digits of $(1919)^{1919}$ is

(1) 56

(2) 63

(3) 45

(4) 54

Answer (2)

Sol. $(1920 - 1)^{1919} = {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + \ldots\ldots$

$\ldots. \; {}^{1919}C_{1918}(1920)^1 - {}^{1919}C_{1919}{}^1$

Last two digits ${}^{1919}C_{1918}(1920) - 1$

$= 3684479$

$\therefore$ Last 2 digits = 79

The product is $7 \times 9 = 63$

7. If $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$, then the value of $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$, then $m + n$ is equal to

(1) 20 (2) 24

(3) 36 (4) 18

Answer (2)

Sol. $\begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$$=\begin{vmatrix}2 & 2 & 2+p+q\\4 & 6 & 8+3p+2q\\6 & 12 & 20+6p+3q\end{vmatrix}+\underbrace{\begin{vmatrix}2 & p & 2+p+q\\4 & 2p & 8+3p+2q\\6 & 3p & 20+6p+3q\end{vmatrix}}_{=0}$$

$$=2\times 2\begin{vmatrix}1 & 1 & 2+p+q\\2 & 3 & 8+3p+2q\\3 & 6 & 20+6p+3q\end{vmatrix}$$

$C_3 \to C_3 \to pC_2$

$$=4\begin{vmatrix}1 & 1 & 2+q\\2 & 3 & 8+2q\\3 & 6 & 20+3q\end{vmatrix}=4\begin{vmatrix}1 & 1 & 2\\2 & 3 & 8\\3 & 6 & 20\end{vmatrix}+0$$

$$=4\begin{vmatrix}1 & 1 & 1\\2 & 3 & 4\\3 & 6 & 10\end{vmatrix}$$

= 8 (1(6) –1(8) + 1(3))

= 8

$$|\text{adj}(\text{adj}(3A))|=(|3A|)^{2^2}=|3A|^4$$

$$=(3^3|A|)^4=3^{12}\cdot|A|^4$$

$$=3^{12}\cdot(2^3)^4$$

$$=3^{12}\cdot 2^{12}$$

8. If $f(x) = x - 1$ and $g(x) = e^x$ and

$\frac{dy}{dx}=\left(e^{-2\sqrt{x}}g(f(f(x)))-\frac{y}{\sqrt{x}}\right)$, where $y(0) = 0$.

Then $y(1)$ equals to

(1) $\frac{2e-1}{e^4}$ (2) $\frac{e-1}{e^4}$

(3) $\frac{e^3-1}{e^4}$ (4) $\frac{e^2-1}{e^4}$

Answer (2)

Sol. $\frac{dy}{dx}=e^{-2\sqrt{x}}e^{x-2}-\frac{y}{\sqrt{x}}$

$$\frac{dy}{dx}=e^{x-2\sqrt{x}-2}\ \frac{-y}{\sqrt{x}}$$

$$\frac{dy}{dx}+\frac{y}{\sqrt{x}}=e^{x-2\sqrt{x}-2}$$

$$J.F=e^{\int\frac{1}{\sqrt{x}}dx}$$

$$=e^{2\sqrt{x}}$$

$$y\cdot e^{2\sqrt{x}}=\int e^{2\sqrt{x}}\cdot e^{-2\sqrt{x}}e^{x-2}dx$$

$$y\cdot e^{2\sqrt{x}}=\int e^{x-2}dx$$

$$ye^{2\sqrt{x}}=e^{x-2}+c$$

$Y(0) = 0$

$$0=\frac{1}{e^2}+c$$

$$\boxed{c=-\frac{1}{e^2}}$$

$\therefore\ ye^{2\sqrt{x}}=e^{x-2}-\frac{1}{e^2}$

$\therefore$ Put $x = 1$

$$ye^2=e^{-1}-\frac{1}{e^2}$$

$$y=\frac{1}{e^3}-\frac{1}{e^4}$$

$$\boxed{y=\frac{e-1}{e^4}}$$

9. Consider two statements

Statement 1: $\lim_{x\to 0}\frac{\tan^{-1}x+\ln\sqrt{\frac{1+x}{1-x}}-2x}{x^5}=\frac{2}{5}$

Statement 2: The $\lim_{x\to 1} x^{\left(\frac{2}{1-x}\right)}$ is equal to e^2 and can be solved by the method $e^{\lim_{x\to 1} f(x)}(g(x)-1)$

(1) Only Statement 1 is true

(2) Only Statement 2 is true

(3) Both Statement 1 and Statement 2 true

(4) Both Statement 1 and Statement 2 False

Answer (1)

Sol. $\lim_{x\to 0}\left(\dfrac{\tan^{-1}x+\ln\sqrt{\dfrac{1+x}{1-x}}-2x}{x^5}\right)$

$$\Rightarrow \lim_{x\to 0}\frac{\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+...\right)+\frac{1}{2}\left(+x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}+...\right)-\frac{1}{2}\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}+...\right)-2x}{x^5}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(x+\frac{1}{2}(x+x)-2x\right)+x^3\left(\frac{-1}{3}+\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{1}{3}\right)+x^5\left(\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}\right)+...}{x^5}$$

$$\Rightarrow \lim_{x\to 0}\frac{\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}\right)x^5}{x^5}=\frac{2}{5}$$

$$\Rightarrow \lim_{x\to 1}x^{\left(\frac{2}{1-x}\right)}=\lim_{x\to 1}\left([1+(x-1)]^{\frac{1}{x-1}}\right)^{\frac{(x-1)2}{(1-x)}}$$

$$==e^{-2}=\frac{1}{e^2}$$

10. Value of $\cot^{-1}\left(\dfrac{\sqrt{1+\tan^2 2}+1}{\tan 2}\right)-\cot^{-1}\left(\dfrac{\sqrt{1+\tan^2 2}-1}{\tan 2}\right)$ is

(1) $\frac{\pi}{2}+\frac{5}{2}$ (2) $\frac{\pi}{2}-\frac{3}{2}$

(3) $2-\frac{\pi}{2}$ (4) $3+\frac{\pi}{2}$

Answer (3)

Sol. $\cot^{-1}\left(\dfrac{\sqrt{1+\tan^2 2}+1}{\tan 2}\right)-\cot^{-1}\left(\dfrac{\sqrt{1+\tan^2 2}-1}{\tan 2}\right)$

$$=\cot^{-1}\left(\frac{|\sec 2|+1}{\tan 2}\right)-\cot^{-1}\left(\frac{|\sec 2|-1}{\tan 2}\right)$$

$$=\cot^{-1}\left(\frac{1-\sec 2}{\tan 2}\right)-\cot^{-1}\left(\frac{\sec 2-1}{\tan 2}\right)$$

$$=\cot^{-1}\left(\frac{\cos 2-1}{\sin 2}\right)-\left(\pi-\cot^{-1}\left(\frac{\sec 2-1}{\tan 2}\right)\right)$$

$$=\left(\pi-\cot^{-1}\left(\frac{1-\cos 2}{\sin 2}\right)\right)-\left(\pi-\cot^{-1}\left(\frac{1+\cos 2}{\sin 2}\right)\right)$$

$$=\cot^{-1}\left(\frac{1+\cos 2}{\sin 2}\right)-\cot^{-1}\left(\frac{1-\cos 2}{\sin 2}\right)$$

$$=\tan^{-1}\left(\frac{\sin 2}{1+\cos 2}\right)-\tan^{-1}\left(\frac{\sin 2}{1-\cos 2}\right)$$

$$=\tan^{-1}\left(\frac{\frac{\sin 2}{1+\cos 2}-\frac{\sin 2}{1-\cos 2}}{1+\frac{\sin 2}{1+\cos 2}\cdot\frac{\sin 2}{1-\cos 2}}\right)$$

$$=\tan^{-1}\left(\frac{-2\cos 2\cdot\sin 2}{1-\cos^2 2+\sin^2 2}\right)$$

$$=\tan^{-1}\left(\frac{-2\sin 2\cdot\cos 2}{2\sin^2 2}\right)$$

$$=\tan^{-1}(-\cot 2)$$

$$=-\tan^{-1}(\cot 2)$$

$$=-\tan^{-1}\left(\tan\left(\frac{\pi}{2}-2\right)\right)=2-\frac{\pi}{2}$$

11. Let $\vec{a}=\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}$ is a vector perpendicular to $\vec{a}$ and lies in the plane of $\vec{a}$, $\vec{b}$ is equal to

(1) $\hat{i}+\hat{j}+\hat{k}$ (2) $-\hat{i}+\hat{j}-\hat{k}$

(3) $\hat{i}-\hat{j}+\hat{k}$ (4) $-\hat{i}-\hat{j}-\hat{k}$

Answer (2)

Sol. $\vec{c}=\vec{a}+\lambda\vec{b}$

$\vec{c}\cdot\vec{a}=0$

$\Rightarrow$ $0 = 26 + \lambda\cdot(2 + 12 + 12)$

$\Rightarrow$ $\lambda = -1$

$\therefore$ $\vec{c}=\vec{a}-\vec{b}$

$=-\hat{i}+\hat{j}-\hat{k}$

12. Area of the region

$\left\{(x,y): 0 \le y \le \sqrt{9x}, y^2 \ge 3-6x\right\}$ (in square units)

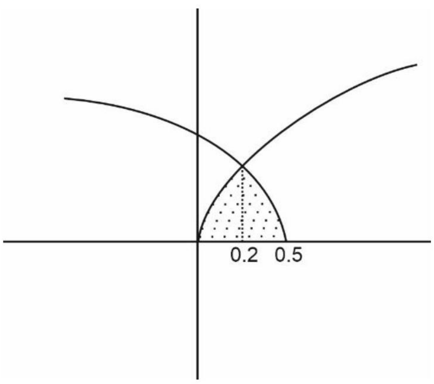
(1) $\frac{1}{3}\left(\frac{9}{5}\right)^{1/2}$ (2) $\frac{3}{5}\left(\frac{8}{5}\right)^{1/2}$

(3) $\frac{1}{3}\left(\frac{7}{5}\right)^{1/2}$ (4) $\frac{1}{9}\left(\frac{7}{5}\right)^{1/2}$

Answer (1)

Sol. $0 \le y \le \sqrt{9x}$

$y^2 \le 3 - 6x$



$$\text{Area} = \int_0^{\frac{1}{5}} \sqrt{9x}\,dx + \int_{1/5}^{1/2} \sqrt{3-6x}\,dx$$

$$\left.\frac{2(9x)^{3/2}}{3\times 9}\right]_0^{1/5} + \left.\frac{2(3-6x)^{3/2}}{3(-6)}\right]_{1/5}^{1/2}$$

$$\frac{2}{27}\times\left(\frac{9}{5}\right)^{3/2} + \frac{1}{9}\left(3-\frac{6}{5}\right)^{3/2}$$

$$= \frac{2}{27}\times\left(\frac{9}{5}\right)^{3/2} + \frac{1}{9}\left(\frac{9}{5}\right)^{3/2}$$

$$= \left(\frac{9}{5}\right)^{3/2}\left[\frac{2}{27}+\frac{1}{9}\right] = \frac{5}{27}\times\left(\frac{9}{5}\right)^{3/2}$$

$$= \frac{1}{3}\times\left(\frac{9}{5}\right)^{1/2}$$

13. Let f be defined as R–{0} $\to R$ such that $f(x) = \frac{x}{3} + \frac{3}{x} + 3$.

If $f(x)$ is strictly increasing in $(-\infty, \alpha_1) \cup (\alpha_2, \infty)$ and strictly decreasing in $(a_3, \alpha_4) \cup (\alpha_4, \alpha_5)$ then $\sum_{i=1}^{5} (\alpha_i)^2$ is equal to

(1) 28 (2) 36

(3) 48 (4) 40

Answer (2)

Sol. $f(x) = \frac{x}{3} + \frac{3}{x} + 3,\ x \ne 0$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2} = \frac{x^2-9}{3x^2}$$

\+ – – +

–3 0 –3

$(-\infty, -3) \cup (0, \infty)$ $f(x)$ increasing

$f(x)$ is decreasing $(-3, 0) \cup (0, 3)$

$(-\infty, \alpha_1) \cup (\alpha_2, \infty) \Rightarrow \alpha_1 = -3, \alpha_2 = 3$

$(-3, 0) \cup (0, 3) \Rightarrow \alpha_3 = -3, \alpha_4 = 0, \alpha_5 = 3$

$$\sum_{i=1}^{5} \alpha_i^2 = (-3)^2 + 3^2 + (-3)^2 + 0^2 + 3^2 = 4.3^2 = 4 \times 9 = 36$$

14. Let A = {0, 1, 2, 3, 4, 5, 6}. Let R_1 be a relation defined on $A \times A$ such that R_1 = {(x, y): max (x, y) $\in$ {3, 4}}.

Consider two statements

Statement 1 : Total number of elements in R_1 is 18

Statement 2: R is symmetric but not reflexive and transitive

(1) Statement 1 is true but Statement 2 is false

(2) Statement 2 is true but Statement 1 is false

(3) Statement 1 and Statement 2 are true

(4) Neither Statement 1 nor Statement 2 are true

Answer (2)

Sol. Let max (x, y) = 3

$\Rightarrow$ {(0, 3), (1, 3), (2, 3), (3, 3), (3, 0), (3, 1), (3, 2)} similarly

Let max (x, y) = 4

$\Rightarrow$ {(0, 4), (1, 4), (2, 4), (3, 4), (4, 4), (4, 0), (4, 1), (4, 2), (4, 3)} Total 16 elements

R_1 is not reflexive : max $(x, y) \in \{3, 4\} \Rightarrow R_1$ is symmetric

$\Rightarrow$ max (y, x) $\in$ (3, 4)

Max (x, y) $\in$ {3, 4} and max (y, z) $\in$ {3, 4} $\Rightarrow$ max = {x, z} $\Rightarrow$ not transitive (0, 4) $\in R_1$ and (4, 0) $\in R_1 \Rightarrow$ (0, 0) $\notin R_1$

15. Let $I_1 = \int_{-\frac{1}{2}}^{1} 2xf\left(2x(1-2x)dx\right)$ and

$I_2 = \int_{-1}^{1} f\left(x(1-x)\right)dx$ then $\frac{I_2}{I_1}$ equals to

(1) 4 (2) 2

(3) 3 (4) 1

Answer (1)

Sol. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf\left(2x(1-2x)dx\right)$

$$I_1 = \int_{-\frac{1}{2}}^{1} 2\left(\frac{1}{2}-x\right)f\left(2\left(\frac{1}{2}-x\right)\left(1-2\left(\frac{1}{2}-x\right)\right)\right)dx$$

$$I_1 = \int_{-\frac{1}{2}}^{1}(1-2x)f\left((1-2x)(2x)\right)dx$$

$$I_1 = \int_{-\frac{1}{2}}^{1} f\left((1-2x)(2x)\right)dx - \underbrace{\int_{-\frac{1}{2}}^{1} 2x\,f\left((1-2x)(2x)\right)dx}_{I_1}$$

$$2I_1 = \int_{-\frac{1}{2}}^{1} f\left((1-2x)(2x)\right)dx$$

Put $2x = t$

$2dx = dt$

$$dx = \frac{dt}{2}$$

$$2I_1 = \frac{1}{2}\int_{-1}^{2} f\left((1-t)(t)\right)dt$$

$$I_1 = \frac{1}{4}\int_{-1}^{2} f\left((1-x)(x)\right)dx$$

$$I_1 = \frac{1}{4}I_2$$

$$4 \Rightarrow \frac{I_2}{I_1}$$

16. A circle touches the parabola $y^2 = 9x$ at (4, 6) and positive x-axis. Find the radius of the circle.

(1) $\frac{20}{3}$ (2) $\frac{10}{3}$

(3) $\frac{1}{3}$ (4) $\frac{5}{3}$

Answer (2)

Sol. Equation of tangent at (4, 6) using $T = 0$

$$y \cdot 6 = \frac{9}{2}(x+4)$$

$\Rightarrow$ $3x - 4y + 12 = 0$

Equation of circle

$(x-4)^2 + (y-6)^2 + \lambda(3x - 4y + 12) = 0$

It touches x-axis.

$\therefore$ put $y = 0$ and make $D = 0$

$(x-4)^2 + 36 + \lambda(3x + 12) = 0$

$\Rightarrow$ $x^2 + (3\lambda - 8)x\ 52 + 12\lambda = 0$

$D = 0$

$(3\lambda - 8)^2 - 4(12\lambda + 52) = 0$

$\Rightarrow \lambda = 12, \frac{-4}{3}$

For $\lambda = \frac{-4}{3}$, circle touches positive x-axis.

$$(x-4)^2 + (y-6)^2 - \frac{4}{3}(3x - 4y + 12) = 0$$

$$r = \frac{10}{3}$$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The sum of squares of roots of $|x-2|^2 - |x-2| - 2 = 0$ and $x^2 - 2|x-3| - 5 = 0$ equals to

Answer (42)

Sol. $|x-2|^2 - |x-2| - 2 = 0$

Let $|x-2| = t$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$t(t-2) + 1(t-2) = 0$

$(t+1)(t-2) = 0$

$t = -1, t = 2$

$\therefore$ $|x-2| = -1,$ $|x-2| = 2$

Number + possible $\Rightarrow x-2=\pm 2$

$\Rightarrow$ $x = 0, 4$

$x^2 - 2|x-3| - 5 = 0$

when $x < 3$

$x^2 + 2(x-3) - 5 = 0$

$x^2 + 2x - 11 = 0$

$x = \dfrac{-2 \pm \sqrt{4+44}}{2}$

$x = \dfrac{-2 \pm 4\sqrt{3}}{2}$

$x = -1 \pm 2\sqrt{3}$

When $x \geq 3$

$\Rightarrow$ $x^2 - 2x + 6 - 5 = 0$

$\Rightarrow$ $x^2 - 2x + 1 = 0$

$\Rightarrow$ $(x-1)^2 = 0$

$\Rightarrow$ $x = 1$ (rejected)

$\therefore$ Sum of square of roots

$= 4^2 + \left(-1+2\sqrt{3}\right)^2 + \left(-1-2\sqrt{3}\right)^2$

$= 16 + 1 + 12 + 1 + 12 = 42$

22.

23.

24.

25.